Nziu and Masu *International Journal of Mechanical and Materials Engineering* (2019) 14:4
https://doi.org/10.1186/s40712-019-0100-y

International Journal of Mechanical and Materials Engineering

ORIGINAL PAPER Open Access

Cross bore size and wall thickness effects on elastic pressurised thick cylinders

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Abstract

Three-dimensional finite element analyses were performed on closed-ended thick-walled cylinders with a radial cross bore under internal pressure. The aim of this study was to determine the behaviour of the hoop stress as well as to establish the optimal Stress Concentration Factors (SCF). Cylinders of thickness ratios of 3.0 down to 1.4 with cross bore size ratios (cross bore to main bore ratio) ranging from 0.1 to 1.0 were studied. The maximum hoop stress was found to increase with the increase in the cross bore size. Amongst the five different circular radial cross bore size ratios studied, the smallest cross bore size ratio of 0.1, gave the lowest hoop stress while the highest stress occurred with a cross bore size of 1.0. Moreover, the lowest SCF occurred in the smallest cross bore size ratio of 0.1 at a thickness ratio of 2.25 with a SCF magnitude of 2.836. This SCF magnitude indicated a reduction of pressure-carrying capacity of 64.7% in comparison to a similar plain cylinder without a cross bore.

Keywords: High-pressure vessels, Cross bore size, Thickness ratios, Finite element analysis, Hoop stress, Stress concentration factor

Introduction

Majority of industrial processes use various types of high-pressure vessels such as boilers, air receivers, heat exchangers, tanks, towers, condensers, reactors etc. in their operation. Failure of high-pressure vessels does occur, being the source of approximately 24.4% of the total industrial accidents in these industries (Nabhani, Ladokun, and Askari, 2012). These failures have resulted in loss of human life, damage of property, environmental pollution, and in some instances, lead to the emergency evacuation of residents living in the surrounding areas (Nabhani et al. 2012). Failure of these vessels is caused by induced stresses in the walls of cylinders resulting from varying operating pressures and temperatures (Ford, Watson, and Crossland, 1981; Iwadate et al. 1981). The induced stresses lead to the formation of stress-related failures in the material such as fatigue, creep, embrittlement, and stress corrosion cracking (Nabhani et al. 2012).

To prevent pressure vessel failures, pressure vessel designers started to use pressure vessel design codes (Kharat and Kulkarni, 2013). However, these codes only give sets of wall thicknesses and their corresponding hoop stresses that are below the allowable working stresses without any detailed stress analysis (Kihiu, 2002). This practice has led to the use of high safety factors in pressure vessel design ranging from 2 to 20 (Masu, 1997). This phenomenon results in an uneconomical use of material which translates into high manufacturing cost of pressure vessels. Other processes such as autofrettage and shakedown (Li, Johnston, and Mackenzie, 2010) are also performed at the manufacturing stage of pressure vessels to increase their strength (Masu and Craggs, 1992). However, it is likely that a more detailed stress analysis will obviate the need for autofrettage, with the accompanying reduction in the manufacturing cost.

Most of the fatigue failures that occur in high-pressure vessels are mainly due to the high magnitude of the hoop stress concentration factor (Masu, 1997), amongst other factors. It is, therefore, necessary to design for an

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optimal minimum hoop stress concentration in order to reduce the occurrence of fatigue failures.

A common form of pressure vessel in use is the thick-walled cylinder with closed ends. It is frequently desirable to provide openings also known as cross bores into the pressure vessels. These cross bores are made either in the end plate or through the vessel wall. Whenever cross bore is provided through the vessel wall, it introduces geometric discontinuities to the cylinder configurations and hence a stress singularity locus. This results in a major source of weakness and a reduction in the pressure-carrying capacity of the vessel below that of a plain cylinder (Masu and Craggs, 1992). The degree of weakness majorly depends on the cross bore geometry configuration such as size, shape, location, obliquity, and cylinder thickness ratios (Makulsawatudom, Mackenzie, and Hamilton, 2004).

Cross bores are referred to as radial when they are drilled at the centre axis of the vessel. On the other hand, cross bores are referred to as offset when drilled at any other chord away from the vessel centre axis (Makulsawatudom et al. 2004). Cross bores are of different shapes and sizes. The common cross bore shapes are circular and elliptical (Nagpal, Jain, and Sanyal, 2012). Whereas, their size ranges from small drain nozzle to large handholds and manholes such as tee junctions (Masu, 1994). A proper understanding of the severity of the stress as a result of each of these cross bore geometry configuration would lead to usage of less conservative safety factors in the design of such vessels, improved plant availability and enhanced safety (Masu and Craggs, 1992).

The effects of the circular cross bore size on a cylinder of a thickness ratio of 1.4 have been cited (Masu, 1994). From the analysis, an optimal size of a circular cross bore is predicted for that particular thickness ratio. This behaviour contradicts other observations (Fessler and Lewin, 1956; Iwadate et al. 1981) that the stress concentration in a plate with a circular hole, decreases with increasing hole size due to the fact that there is less abrupt change in cross-sectional area in a large hole. Hence, there is a need for further research into the effect of the optimal size of a circular cross bore on various thickness ratios to exploit any potential benefits.

Several techniques, namely, experimental, analytical and numerical (also known as computational), are used in the analysis of stress concentration in high-pressure vessels. Numerical methods use packages such as finite element analysis (FEA), finite difference, finite volume, boundary integral element (BIE) and mesh-free methods for stress analysis (Masu, 1989; Nagpal et al. 2012).

FEA numerical method has been extensively used for stress analysis compared to experimental and analytical methods (Kharat and Kulkarni, 2013). This is due to its ability to perform simulation on complex geometries. The FEA method also gives fairly accurate results (Wang et al. 2012) compared to its competitors (experimental and analytical methods). Various techniques are used to develop governing equations for solving FEA problems. These techniques include equilibrium and compatibility equations, nodal displacement, variational and principle of virtual work.

Methodology

The aim of this study was to determine the behaviour of the hoop stress as well as to establish the optimal stress concentration factors (SCF). Seven cylinders with various wall thickness ratios and cross bore sizes were studied. The wall thickness ratio of the cylinders (K) was selected to coincide with those discussed in the reviewed literature by Makulsawatudom et al. (2004), Masu (1989, 1991), and Nihous, Kinoshita, and Masutani (2008). These included K values of 1.4, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0. A total of five different radial cross bores, comprising of both small and large cross bores were studied. The cross bore size ratios (cross bore to main bore ratio) of 0.1, 0.3 and 0.5 were classified as small cross bores (Steele, Steele, and Khathlan, 1986). Whereas, the cross bore size ratios of 0.7 and 1.0 (pipe junction) were categorised as large cross bores.

A configuration of a radial circular cross bore in a thick-walled cylinder is shown in Fig. 1.

Where

p_i is the internal pressure.

σ_{θ_1} is the hoop stress generated by the pressurised main bore.

σ_{r_1} is the radial stress generated by the pressurised main bore.

Three-dimensional finite element analysis

Finite element analyses were performed on the high-pressure vessels with a radial cross bore using a commercial software programme called Abaqus version 6.16. Owing to the symmetrical configuration of the pressure vessel, only an eighth of the structure was analysed. A total of 35 different part models were created and analysed. A three-dimensional deformable solid body was created by sketching an eighth profile of the pressure vessel face. The face of the pressure vessel was then extruded to form the depth of the cylinder. The depth of the cylinder was long enough to mitigate the effects of the closed ends' closures of the

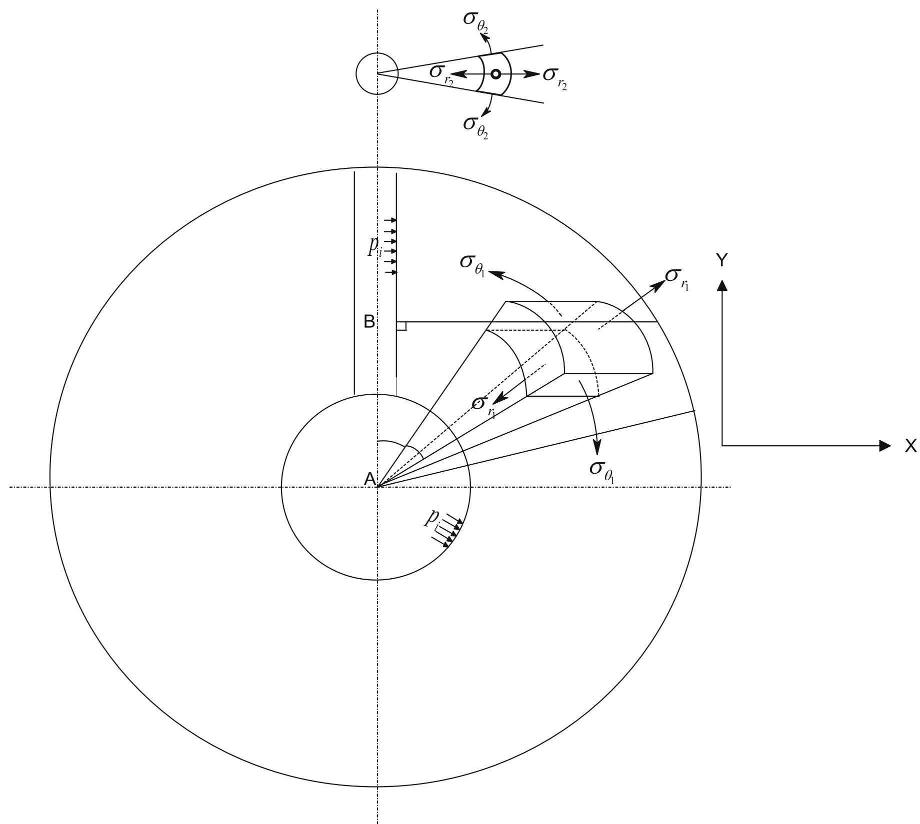

Fig. 1 Main bore—cross bore configuration

cylinder vessels were restricted from being transmitted to the other far end of the cylinder. One of the model profiles created at this stage is shown in Fig. 2.

For this stress analysis problem, a linear elastic model with material properties as indicated in Table 1 was assumed throughout in the modelling process. The material properties chosen for this simulation were similar to those commonly used in the technical literature of high-pressure vessels (Chaudhry, Kumar, Ingole, Balasubramanian, and Muktibodh, 2014; Choudhury, Mondol, and Sarkar, 2014).

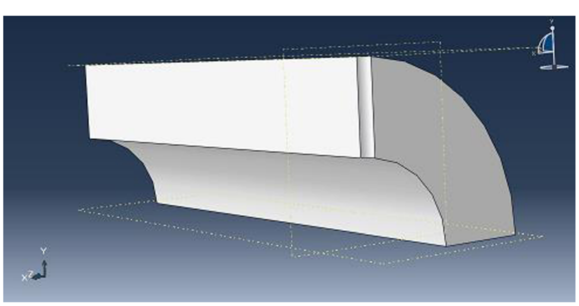

Fig. 2 Selected profile of the model

The section properties of the model were defined as being solid and assigned to the entire profile previously created in Fig. 2. This action was then followed by the creation of a single assembly. A single assembly usually contains all the geometry in the finite element model. This procedure allowed the creation of a part instance that is independent of the mesh. The model was then oriented in line with the global Cartesian coordinates axes, i.e. X, Y and Z axes.

The analysis to be used for this simulation was configured by creating a static pressure step. It is worthwhile to mention that the application of different types of loads and boundary conditions depend on the total number of analysis steps created. To prevent any rigid movement of the model, symmetry boundary conditions were applied at the three cut sections of

Table 1 Material properties for the static analysis

Parameter	Value
Young's modulus of elasticity	190 GPa
Poisson's ratio	0.29
Density	7800 kg/m^3

the cylinder. These symmetry boundary conditions were applied at cut regions in X, Y and Z axes.

The pressure vessel was loaded with an internal pressure at both the main bore and the cross bore. The internal pressure was taken as 1 MPa, being the most common pressure used in pressure vessel analyses (Gerdeen, 1972). In addition, a uniform axial stress σ_z, calculated by using Eq. 1 for each thickness ratio, was applied at the far end of the vessels. This axial stress simulated the end effects generated by the closed end closures in the pressure vessels.

$$\sigma_z = \frac{p_i}{K^2-1} \qquad (1)$$

Where

P_i is the internal pressure.

K is the thickness ratio

The magnitude of axial stress calculated for each thickness ratio is tabulated in Table 2.

The model meshing was done by dividing the model into small geometrical sections. The mesh around the cross bore region was biased by increasing the number of elements, commonly referred to as mesh density. The size of the element chosen ranged from 0.003 m to 0.004 m. This high mesh density around the cross bore region increased the capture of the localised stress concentration (Fagan, 1992).

According to the Abaqus software guideline, only second-order hexahedral and tetrahedral elements are recommended for stress concentration problems. Thus, a 20-noded second-order, C3D20R hexahedral (brick) isoparametric elements were used in cylinders with the following cross bore to main bore ratios: 0.1, 0.3, 0.5 and 0.7. On the other hand, second-order C3D10 tetrahedral elements with 10 nodes were used for pipe junction models since they are less sensitive to the initial shape of the element; therefore, their vulnerability to distortion is low (Fagan, 1992). A meshed profile of one of the model part is shown in Fig. 3.

Principally, the accuracy of the results depends on the quality of the mesh and its density. In this study, the degree of the accuracy of the results as well as the mesh convergence was confirmed by comparing the obtained FEA results, with their corresponding analytical results in areas far away from the cross bore (Masu, 1989). It was assumed that the effects of the cross bore are limited only to the area surrounding it, usually about 2.5 cross bore diameters (Masu, 1989).

Table 2 Axial stresses

K	1.4	1.5	1.75	2.0	2.25	2.5	3.0
σ_z (MPa)	1.04166	0.80	0.485	0.333	0.246	0.190	0.111

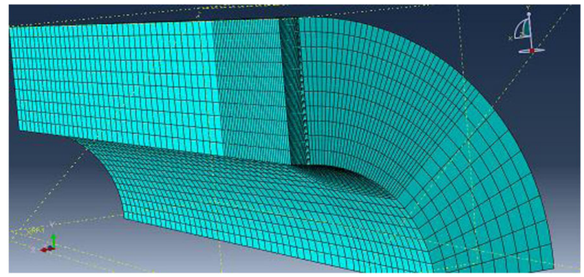

Fig. 3 Selected profile of the mesh

The results of this modelling were presented per unit pressure for ease of the comparisons, since the analysis was performed under elastic conditions.

Results and discussion

Effects of cross bore size and cylinder thickness ratio on maximum hoop stress

Figures 4 and 5 show the variation of hoop stress with cross bore size and thickness ratio, respectively.

As shown in Fig. 4, it was observed that the hoop stress increases with the increase in the cross bore size. This increase in the hoop stress was more pronounced in the cylinders with small thickness ratios, specifically $K = 1.4$ and 1.5. For instance, the comparison of hoop stress between the cross bore size ratio of 0.1 and that of 1.0 for $K = 1.4$ and 1.5 gave a hoop stress increase of 180.14% and 167.64%, respectively. The structural stiffness of the cylinder is affected by both the thickness ratio and cross bore size. As the thickness ratio reduces, the structural stiffness reduces leading to high magnitudes of hoop stresses. Further reduction of the structural stiffness is observed whenever there is an increase in the cross bore size leading to much higher magnitudes of the hoop stresses.

For a similar cross bore size ratio, the difference in hoop stresses for $K = 2.5$ and 3.0, showed a hoop stress increase of 60.94% and 32.95%, respectively. These observations revealed that the rise in hoop stress due to the size of the cross bore reduces with increase in thickness ratio. Likewise, the structural stiffness of the cylinder increases with increase in thickness ratio leading to lower magnitudes of hoop stresses in comparison to the cylinder with smaller thickness ratios.

Figure 5 further confirmed the earlier finding that the magnitude of hoop stress in a cross bored cylinder reduces as the thickness ratio increases. The increase in hoop stress due to the radial cross bore was highest in $K = 1.4$. The hoop stress increase in $K = 1.4$ between the cross bore size ratios of 0.3, 0.5, 0.7 and 1.0 in reference to 0.1 was 27.46%, 72.68%,

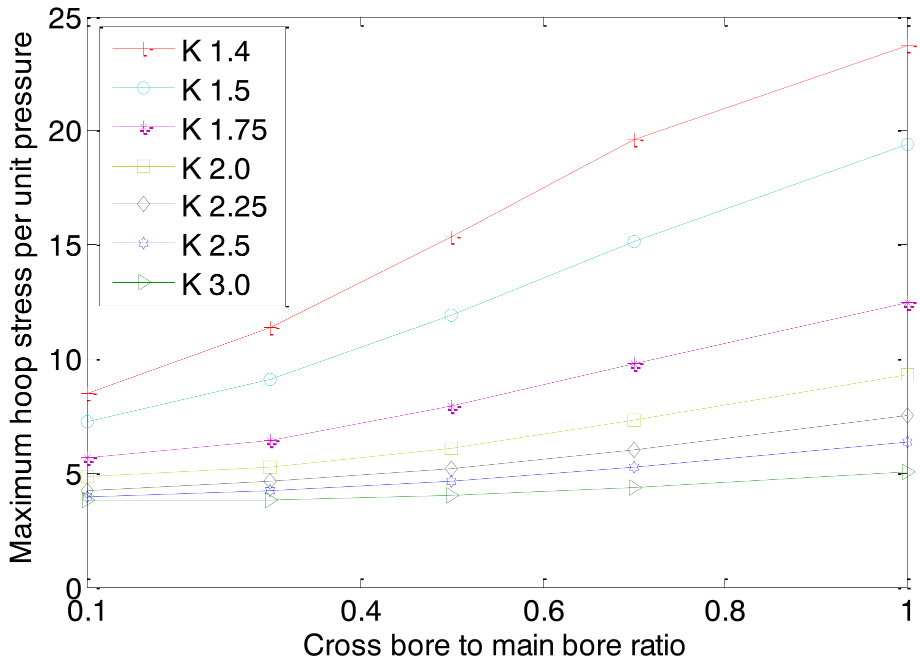

Fig. 4 Hoop stress vs cross bore size

120.394% and 166.17%, respectively. In contrast, the rise in hoop stress was observed to reduce gradually with an increase in the thickness ratio. The lowest rise in hoop stress was reported at $K = 3$ since, at this value, the rise in hoop stress between cross bore size ratio of 0.1 in comparison to that of the cross bore size ratios of 0.3, 0.5 and 0.7 was below 14.34%. Only, a rise of 32.96% in hoop stress between similar cross bore sizes as indicated in the preceding paragraph was noted.

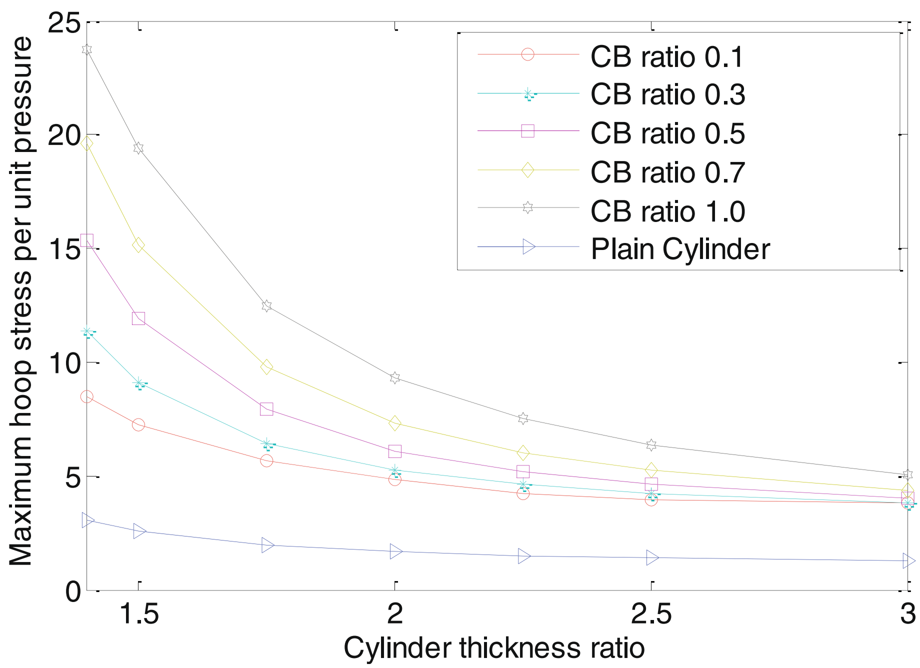

Fig. 5 Hoop stress vs thickness ratio

The magnitude of hoop stress in a cylinder with radial cross bore was higher in comparison to a similar plain cylinder without a cross bore as indicated in Fig. 5. This observation was in line with other previous studies done by Comlekci, Mackenzie, Hamilton, and Wood (2007), Makulsawatudom et al. (2004) and Masu (1997).

Noticeably, the cross bore size ratio of 0.1 gave the lowest rise in hoop stress when compared to the other four cross bore sizes. Further comparison was done between the hoop stresses generated by the cross bore size ratio of 0.1 and that of the plain cylinder without the cross bore. This comparison established the behaviour of stress variation in reference to the plain cylinder. The rise in hoop stress for thickness ratios 1.4, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0 was found to be 174.67%, 178.5%, 186.34%, 192.2%, 183.58%,186.97 and 204.4%, respectively, with the lowest and highest rise, occurring at $K = 1.4$ and 3.0 respectively. A significant drop in the hoop stresses between $K = 2.0$ and 2.25 was also observed. This stress drop implied the existence of stress transition point between the two thickness ratios. This is probably an indication of change of state of stress from plane stress to plane strain. Usually, at the transition point, there is stress redistribution around the cross bore that may lead to the stress drops and peaks.

In general, the cross bore size ratios of 0.1 gave the lowest hoop stress, while the highest stress occurred in 1.0. A similar occurrence had been reported by other previous studies Hearn (1999) and Nihous et al. (2008). Usually, large cross bores entail excessive removal of materials in the cylinder, and as a result, only little material is left to bear the applied load. This excessive removal of material leads to an increase in hoop stress which might cause failure of the cylinder.

Location of maximum principal stress in the cylinder

The location of the maximum hoop stress on the cylinder generally occurred along the radial cross bore. The exact positions for all the studied cases are tabulated in Appendix. The maximum hoop stress for most of the studied cases occurred in the transverse plane, at the intersection between the main bore and the cross bore except for the values of $K = 1.4$, 1.5 and 1.75. The location of the maximum hoop stress for $K = 1.4$, 1.5 and 1.75 due to the cross bore size ratio of 0.1 occurred approximately 1.25 mm away from the intersection along the transverse plane. This occurrence was attributed to the change in state of stress from plane stress to plane strain usually resulting from geometry change. The change of state of stress results to stress redistribution around the cross bore causing stress peaks. Similar observations had earlier been reported by Masu (1997).

Generally, the maximum hoop stress occurred along the cross bore transverse plane for all the cross bore size ratios between 0.1 and 0.7. This occurrence signified the existence of uniform stress field distribution around the cross bore indicating plane stress conditions.

With the exception of $K = 1.5$ and 1.75, it was also noted that the location of maximum principal stress due to the largest cross bore size, 1.0, shifted slightly away from the cross bore transverse plane. Upon loading, large cross bores experience varying bending and shearing stresses along cross bore. This occurrence causes non-uniform stress fields around the cross bore leading to the location shift of maximum principal stresses.

Effects of cross bore size and thickness ratio on hoop stress concentration factor

Stress concentration factor (SCF) is a dimensionless quantity that enables effective comparison of stresses between different parameters regardless of their size, shape, thickness or the applied load. In this work, the stress concentration factor was defined as the ratio of localised critical stresses in a cross bore cylinder to the corresponding one in a similar cylinder without a bore. The SCFs for cylinders with different cross bore sizes and thickness ratios were calculated based on locations with the highest magnitudes of hoop stress in the cylinder. Figures 6 and 7 show the variation of stress concentration factor with cross bore size and thickness ratio.

As illustrated in Fig. 6, the hoop stress concentration factor was lowest at smallest cross bore with size ratio of 0.1. Moreover, the lowest SCF given by this cross bore size occurred in $K = 2.25$ with a magnitude of 2.836, while the highest stress concentration factor predicted by the same cross bore size occurred at $K = 3.0$ with a magnitude of 3.044, being an increase of 7.33%. In contrast, the highest SCF in the cylinder was reported in the largest cross bore size of 1.0. In this cross bore size of 1.0, the highest SCF occurred at $K = 1.4$ with the magnitude of 7.687, whereas, the lowest SCF was noted at $K = 3.0$ with a magnitude of 4.047. Generally, it was observed that the magnitude of SCF increased with increase in the cross bore size. As the cross bore size increases, the structural stiffness of the cylinder reduces. This leads to the generation of high hoop stresses and consequently high SCFs.

With the exception of the smallest cross bore size, the SCF was observed to reduce with an increase in the thickness ratio as illustrated in Fig. 7. The rise in magnitude of SCF given by the smallest cross bore of 0.1, in comparison to similar ones of the plain cylinder for $K = 1.4$, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0 which

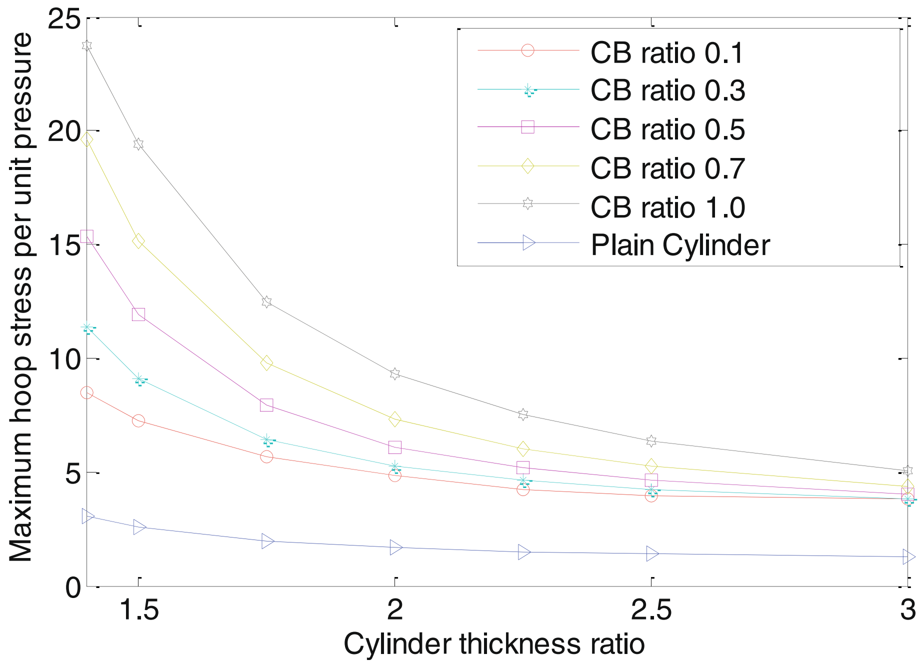

Fig. 6 Hoop SCF vs cross bore size

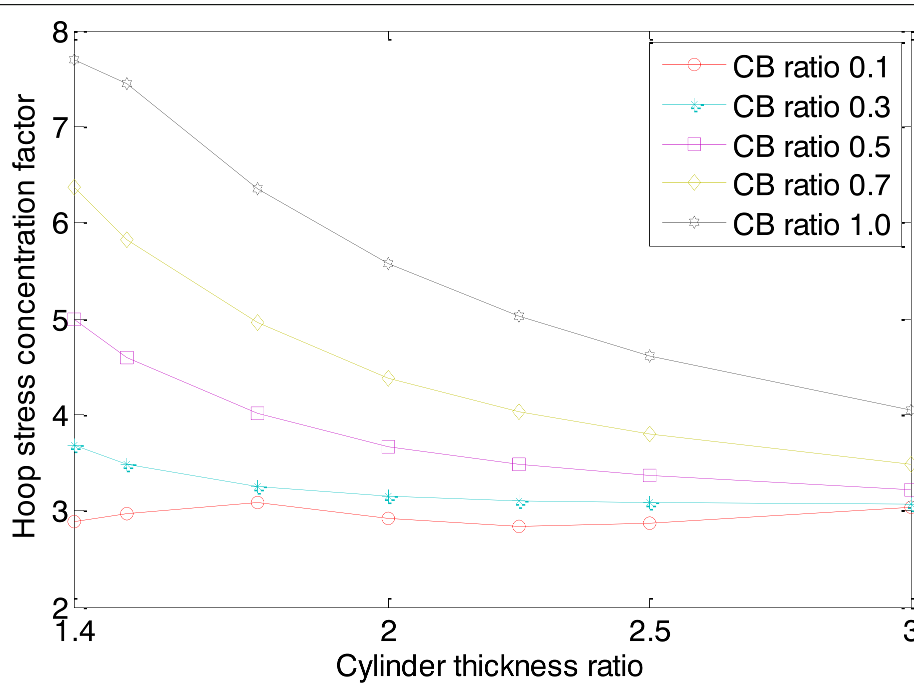

Fig. 7 Hoop SCF vs thickness ratio

Table 3 Comparison of stress concentration factors for radial cross bores

	Comlekci et al. (2007) study	Makulsawatudom et al. (2004) study	Present study	Comlekci et al. (2007) study	Present study
K	CB = 0.1	CB = 0.1	CB = 0.1	CB = 0.25	CB = 0.25
1.4	2.76	–	2.888	3.01	3.482
1.5	2.81	–	2.977	2.95	3.361
1.75	2.93	–	3.078	2.91	3.208
2.0	3.03	2.89	2.922	2.92	3.090
2.25	3.11	–	2.869	2.95	3.036
2.5	3.18	–	3.044	2.98	3.03

were found to be 188.8%, 197.7%, 207.8%, 192.2%, 183.6%,186.9% and 204.4%, respectively. This rise in SCF profile was slightly higher than that of hoop stresses presented earlier in the hoop stress section. The rise in stress value was attributed to the consideration of the location of the maximum principal stress during the computation of the SCF. It is worthwhile noting that the hoop stress does not take into consideration the location effects.

A summary of some of the results from the present work in comparison to the existing published data is indicated in Table 3. Unlike in the other previous studies, the comparison of SCF took into consideration both the cross bore size and the thickness ratio. The results of the hoop SCF presented in this study were compatible with those published by Comlekci et al. (2007) and Makulsawatudom et al. (2004) for the cross bore size ratio of 0.1. Comlekci et al. (2007) published other additional data having different cross bore size ratios. The closest cross bore size ratio to the present study being 0.25. This occurrence necessitated the results of the presented study to be interpolated between the cross bore ratios of 0.1 and 0.3 to give SCF values for 0.25. The values were read from Fig. 4. Upon comparison, the two results were also found to be in close agreement.

Contrary to the solution of the circular hole in a plate, where the stress concentration factor reduces with increasing hole size (Nagpal et al. 2012), and a maximum SCF of 3.0, regardless of the hole size was found, the hoop stress as well as stress concentration factor resulting from the introduction of circular radial cross bores in thick cylinders was found to increase with an increase in the cross bore size. For instance, in the development of the solution of the circular hole in a plate, it was

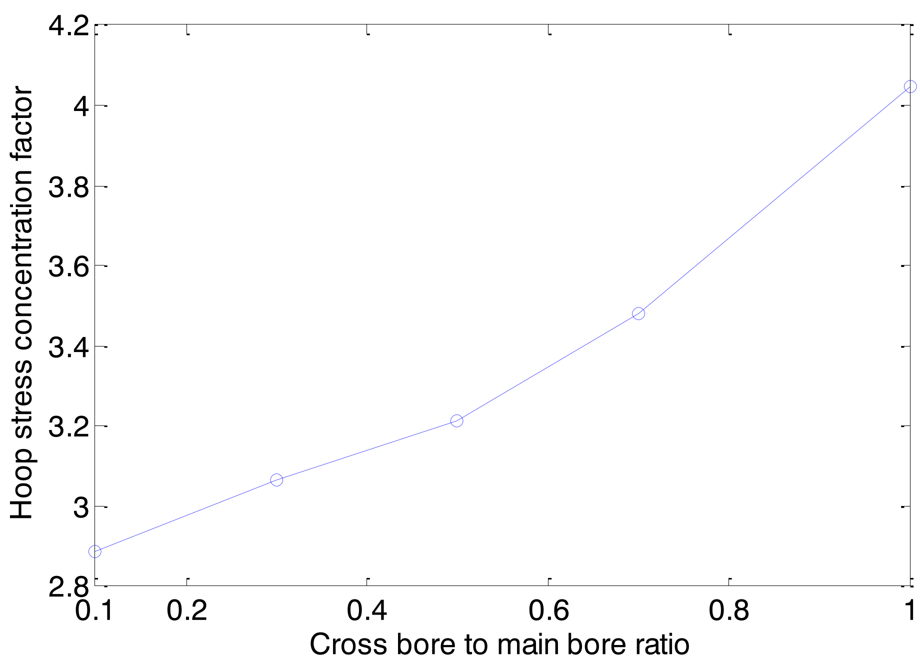

Fig. 8 Optimal SCF vs cross bore size

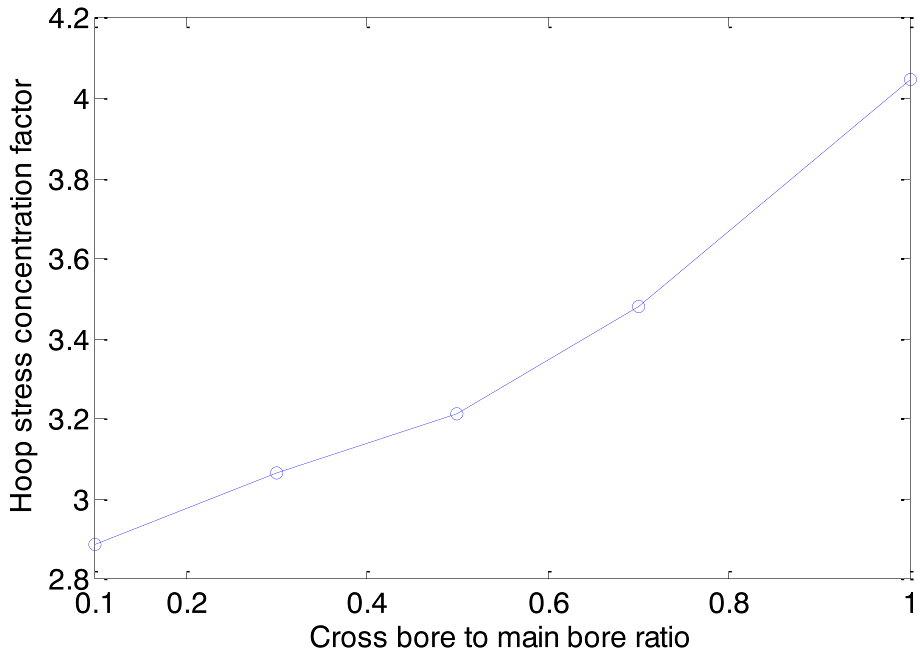

Fig. 9 Optimal SCF vs thickness ratio

assumed that the width of the plate is large in comparison to the cross bore. Further, it was also assumed that the applied load was in the axial position. In addition, the plane stress theory ignores the effects of the shearing stress as well as the bending moment around the cross bore. Thus, it assumes a uniform stress distribution field around the cross bore. These assumptions might not be applicable in thick-walled cylinders due to their curvilinear nature.

Another analytical study published by Faupel and Harris (1957) gave the SCF for a circular hole in a closed thick cylinder as being constant at 2.5 without taking into consideration the size of the cross bore. This analytical solution by Faupel and Harris (1957) was also in contradiction to the findings of this study. In conclusion, the size of a cross bore as well as the thickness ratio plays a significant role in determining the hoop stress as well as the stress concentration factor. These findings are also extended to the fatigue behaviour of the component.

Optimal cross bore sizes and cylinder thickness ratios

Figures 8 and 9 show the optimal SCF curves at each cross bore size ratio and thickness ratio obtained from the studied radial circular cross bores.

Amongst the five cross bore sizes studied, the smallest cross bore size ratio of 0.1 gave the lowest magnitude of SCF of 2.836 at $K = 2.25$. This SCF magnitude indicated a reduction of pressure-carrying capacity by 64.7% in comparison to a similar plain cylinder without a cross bore. This pressure-carrying capacity was slightly higher than 60% cited earlier by Masu's (1989) study. In this regard, therefore, this cross bore size ratio was selected as the optimal size for a radial circular cross bore.

Conclusions

The following conclusions were drawn from the present study:

i. The maximum hoop stress increases with the increase in the cross bore size. Amongst the five different circular radial cross bore size ratios studied in seven cylinders, the smallest cross bore size ratio of 0.1 gave the lowest hoop stress while the highest stress occurred with a cross bore size of 1.0.
ii. Amongst the five different circular radial cross bore ratios studied in seven cylinders, the lowest SCF occurred in the smallest cross bore size ratio of 0.1 when $K = 2.25$ with an SCF magnitude of 2.836. This SCF magnitude indicated a reduction of pressure-carrying capacity of 64.7% in comparison to a similar plain cylinder without a cross bore.

Appendix

Table 4 Location of maximum hoop stress in the cylinder

Thickness	Cross bore size ratio	Radius *R* (m)	Horizontal distance measured from the transverse axis of the main cylinder (m)
1.4	0.1	0.026	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0.001
1.5	0.1	0.02625	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0
1.75	0.1	0.02625	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0
2.0	0.1	0.025	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0.001
2.25	0.1	0.025	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0.001
2.5	0.1	0.025	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0.001
3.0	0.1	0.026	0
	0.3	0.025	0
	0.5	0.025	0
	0.7	0.025	0
	1.0	0.025	0.001

The data is given in the form of the main cylinder radius and the horizontal distance from the transverse edge of the cross bore

Abbreviations

K: Thickness ratio (outer diameter to inner diameter); SCF: Stress concentration factor

Acknowledgements

This research work was supported by Vaal University of Technology. The authors wish to thank the department of Mechanical Engineering at Vaal University of Technology for facilitating this work.

Funding

Not applicable

Availability of data and materials

Although much of the information has been provided in this article, the raw data generated in this work is also available upon request.

Authors' contributions

Both authors contributed to the development of this journal article. Both authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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Received: 30 January 2019 Accepted: 14 March 2019
Published online: 25 April 2019

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