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Investigation of a new methodology for the prediction of drawing force in deep drawing process with respect to dimensionless analysis

Saeed Hajiahmadi, Majid Elyasi* and Mohsen Shakeri

Abstract

In this research, geometric parameters were given in dimensionless form by the Buckingham pi dimensional analysis method, and a series of dimensionless groups were found for deep drawing of the round cup. To find the best group of dimensionless geometric parameters, three scales are evaluated by commercial FE software. After analyzing all effective geometric parameters, a fittest relational model of dimensionless parameters is found. St12 sheet metals were used for experimental validation, which were formed at room temperature. In addition, results and response parameters were compared in the simulation process, experimental tests, and proposed dimensionless models. By looking at the results, it very well may be inferred that geometric qualities of a large scale can be predicted with a small scale by utilizing the proposed dimensionless model. Comparison of the outcomes for dimensionless models and experimental tests shows that the proposed dimensionless models have fine precision in determining geometrical parameters and drawing force estimation. Moreover, generalizing proposed dimensionless model was applied to ensure the estimating precision of geometric values in larger scales by smaller scales.

Keywords: Dimensional analysis, Geometrical parameters, Dimensionless model, Π-Buckingham pi theorem, Deep drawing

Introduction

Process performance and better control of product quality are the main area of active research for the deep drawing process as the complex form of the sheet metal forming process. The quality of the process is still highly dependent on trial and error over large sizes which require elevated production costs. Finite element analysis (FEM) is also used which can be computationally costly and highly dependent on constitutive laws. Because of the simplification of FEM on constitutive laws and boundary conditions, it cannot cover the wide variety of physical activities across the wide range of length scales. Therefore, the experimental tests are essential for verifying FEM results. Also, it is important to select the proper process parameters to achieve flawless parts when the process changes from small to large scale. Although many studies have been done through dimensional analysis, quite a few of them contain the metal forming and articles involving both dimensional analysis and sheet metal forming especially predicting suitable geometrical parameters in different dimensional scales are still pretty limited (Davey et al. 2017; Liu and Yin 2018). It should also be noted that although the scale changes for the generalization of experimental outcomes are not a new challenge for fluid and dynamic applications but no attention has yet been paid to its application in metal forming (Li et al. 2019; Al-Tamimi et al. 2017).

Dimensional analysis plays an important role in evaluating the process on different scales. While this method depends on the complexity of the problem, the most significant benefits of this method are simplicity, no need to understand the fundamental process model, general

* Correspondence: elyasi@nit.ac.ir; elyasima@yahoo.com
Faculty of Mechanical Engineering, Babol Noshirvani University of Technology, Babol, Iran

dimensional model diagnosis, and process trend prediction in different scales (Tan 2011). In this case, Navarrete et al. (2001) demonstrated that the prediction of required force for an open die forging can be presented with less than 15% error rate by the application of dimensional analysis and Π- Buckingham pi theorem. Although the evaluation performed by dimensional analysis was very helpful in predicting force, the friction coefficient and the complexity of the geometry could limit the process of extracting information. Pawelski (1992) reported one of the few explanations of metal forming dimensional analysis. In this research, Buckingham pi theorem was used to define the effect of lubricants on cold rolling. Jamadar and Vakharia (2016) developed a new methodology based on dimensional analysis to measure the localized faults, an improvement of the computational efficiency in the nonlinear dynamic analysis for the rolling contact bearing. A good agreement between proposed theoretical models obtained by dimensional analysis with experimental data precisely shown the validity of the theoretical model. Finally, it was mentioned that the dimensional analysis tool can strongly provide great help for investigating the actual industrial scale by the laboratory studies. Ajiboye et al. (2010) investigated the determination of the friction role in cold forging. They use dimensional analysis to detect friction effects and Buckingham pi theorem for predicting its value. They found that changes in friction trend can be strongly predicted by a linear model obtained from dimensional analysis.

Methods

Aim and design of the study

Although dimensional analysis and the Buckingham pi theorem have been used in the fields of fluids, dynamic analysis, friction, and bulk forming, but regarding previous studies, in the case of reducing time and costs of manufacturing and simulation process of large dimensional scales, there is no attention paid to sheet metal forming so far. This study demonstrates that how the required geometrical parameters for designing and manufacturing of deep drawing process can be made in dimensionless form by Buckingham pi theorem. Moreover, measurement of drawing force and prediction of failure in metal forming processes need accurate and costly instrumentations. Therefore, it can be inferred that exceeding tensile stresses which causes fracture and failure lead to very crucial situations in the area of sheet metal forming. This condition can be successfully controlled by the selection of accurate geometrical parameters for the deep drawing process.

Considering the fact that there is no investigation on the dimensionless parameters for sheet metal forming, the main purpose of the current study is to suggest new dimensionless models for reducing the manufacturing costs by predicting drawing force as the crucial factor to evaluate the flawless quality of the deep drawing process in its original large size by small scale laboratory samples. For this reason, the similarity law and the Buckingham π theory were used as key theories that are widely used in dimensional fields. The geometric dimensional groups of the deep drawing process of round cups are assessed on different scales. Then, the best dimensionless models to predict drawing force at the moment of tearing are determined by stepwise regression method and ANOVA taking into consideration the effective geometric parameters. Also, the accuracy of the dimensionless models was investigated using experimental results; the prediction of the drawing force was presented. Finally, the precision of the proposed dimensionless models and dimensionless analysis was also investigated using the generalization technique. Afterward, the results of experimental tests and the proposed dimensionless model were compared.

Buckingham pi theorem

There are several techniques to reduce the number of dimensional variables to a smaller number of dimensionless groups. The method provided here has been suggested by Buckingham (1914) and is now called the Buckingham pi theorem. The name pi is derived from the mathematical notation π, i.e., the product of variables. The dimensionless groups found in the theorem are denoted by π_1, π_2, π_3, etc. The technique enables pi groups to be discovered in sequential order without the use of free exponents. The first part of the pi theorem describes what to expect in the decrease of variables:

If a physical process satisfies the dimensional homogeneity and involves "n"-dimensional variables, it can be reduced to a relation between only "m" dimensionless variables or pi groups. The Buckingham pi theorem explains that if there are "n"-dimensional variables in a problem, the dimensions or quantities that are related in a homogeneous condition can be described quietly by "m" dimensions (Buckingham 1914; Allamraju and Srikanth 2017).

The most general form of physical equations among a number of "n" physical quantities contains "n" symbols $Q_i \cdots Q_n$, one for each kind of quantity, and also, in general, a number of ratios r', r'', etc., so that it may be written as Eq. (1).

$$f\left(Q_1, Q_2, Q_3 \ldots\ldots\ldots Q_n, r', r'', \ldots\right) = 0 \quad (1)$$

It is supposed by (Buckingham 1914) that the ratios "r" do not vary during the phenomenon. For example, if the equation describes a property of a material system and involves lengths, the system shall remain geometrically similar to itself during any changes in the size which may occur. Under this condition, Eq. (1) reduces to Eq. (2).

$$f(Q_1, Q_2, Q_3 \ldots\ldots\ldots\ldots Q_n) = 0 \quad (2)$$

If none of the quantities involved in the relationship has been overlooked, the equation will give a complete description of the relation subsisting among the quantities represented in it and will be a complete equation. The coefficients of a complete equation are dimensionless numbers, i.e., if the quantity "Q" is measured by an absolute system of units, the coefficients of the equation do not depend on the sizes of the fundamental units but only on the fixed interrelations of the units which characterize the system and differentiate it from any other absolute system. To illustrate what is meant by a "complete equation," the familiar equation is considered as Eq. (3).

$$\frac{pv}{\theta} = \text{constant} \quad (3)$$

In which "p" is the pressure, "v" is the specific volume, and "θ" is the absolute temperature of a mass of gas. The constant is not dimensionless but depends, even for a given gas, on the units adopted for measuring "p," "v," and "θ"; the equation is not complete. Further investigation shows that Eq. (3) can be written as Eq. (4).

$$\frac{pv}{R\theta} = N \quad (4)$$

In which the symbol "R" stands for a quantity characteristic of each gas and differs from one to another, but fixed for any given gas when the units of p, v, and θ are fixed. It was recognized that "R" is a quantity that can be measured by a unit derived from those of "p," "v," and "θ." If we do express the value of "R" in terms of a unit thus derived, N is a dimensionless constant and does not depend on the sizes of the units of "p," "v," and "θ." Therefore, the equation is now a "complete equation." Every complete physical equation has a more specific form as Eq. (5).

$$\sum M Q_1^{b_1} Q_2^{b_2} \ldots Q_n^{b_n} = 0 \quad (5)$$

where "M" is a dimensionless number. All the terms of a physical equation must have the same dimensions, or that every correct physical equation is dimensionally homogeneous (Buckingham 1914). Equation (5) can be divided by any term, and it takes the form of Eq. (6).

$$\sum N Q_1^{a_1} Q_2^{a_2} \ldots Q_n^{a_n} + f\left(Q_1, Q_2, Q_3 \ldots\ldots\ldots\ldots Q_n, r', r'', \ldots\right) = 0 \quad (6)$$

In which "N" is a dimensionless number. Considering the dimensional homogeneity, the exponents a_1, a_2, $\cdots$ a_n, of each term of Eq. (6) must be dimensionless or that Eq. (7) as a dimensional equation is satisfied.

$$Q_1^{a_1} Q_2^{a_2} \ldots Q_n^{a_n} = f\left(Q_1, Q_2, Q_3 \ldots\ldots\ldots\ldots Q_n, r', r'', \ldots\right) \quad (7)$$

Equation (7) can be reduced to Eq. (8) by representing "π" as a dimensionless product.

$$Q_1^{a_1} Q_2^{a_2} \ldots Q_n^{a_n} = \pi \quad (8)$$

Since "π" is dimensionless, "π^x" is dimensionless. Furthermore, any product of the form "$\pi_n^{a_n}$" is also dimensionless. Hence, π_1, π_2 ... π_i represent all the separate independent dimensionless products of Eq. (8) which can be made up under Eq. (7) from the quantities Q. Now, there are, so far as this requirement is concerned, no restrictions on the number of terms, the values of the coefficients, or the values of the exponents. Hence, Eq. (5) may more simply be written as Eq. (9).

$$f(\pi_1, \pi_2, \pi_3 \ldots\ldots\ldots\ldots \pi_i) = 0 \quad (9)$$

Because of the principle of dimensional homogeneity, every complete physical equation of Eq. (2) which is reducible to Eq. (10) can be defined by Eq. (11).

$$[\pi_1] = [\pi_2] = \ldots = [\pi_i] = f\left(Q_1, Q_2, Q_3 \ldots\ldots\ldots\ldots Q_n, r', r'', \ldots\right) \quad (10)$$

The second part of the theorem demonstrates how to discover pi groups at a moment. Discovering pi groups depends on finding the reduction factor "k" and selecting "k" scaling variables which do not form a pi group among themselves. Each desired pi group will be a power product of these "k" variables. Therefore, each pi group found is independent.

The number "k," of separate independent arguments of "f," is the maximum number of independent dimensionless products of Eq. (8) which can be made by combining the n quantities Q_1, Q_2 ... Q_n in different ways. The value of reduction factor "k" can be determined by the factor "n" as the number of arbitrary fundamental units needed as a basis for the absolute system $[Q_1]$, $\cdot\cdot$ $[Q_n]$. It is mentioned by (Buckingham 1914) that there is always at least one set of "m" which may be used as fundamental units, the remaining $(n - m)$ being derived from them. So, the relationship between the "n" quantities can always be described by the reduction factor "k"

and independent π terms. The reduction factor "k" equals the maximum number of variables that do not form a pi group among themselves and is always less than or equal to the number of dimensions describing the variables

Now, each equation of Eq. (7) with a particular set of exponents, "a" is an equation to which the dimensions of the units [Q] are subject. But since (n – m) of the units are derivable from the other "k" and the units are otherwise arbitrary, it is evident that each equation of Eq. (7) is, in reality, equivalent to one of these equations of derivation. There are, therefore, (n – m) equations of Eq. (7) and the number of products for Eq. (2) which appear as independent variables in Eq. (9) is presented by Eq. (11).

$$k = n\text{–}m \tag{11}$$

Furthermore, if $[Q_1]$, $[Q_2] \cdots [Q_m]$ are "m" of the "n" units which might be used as fundamental, the relations for Eq. (5) can be written as Eq. (12).

$$\begin{aligned}
[\pi_1] &= \left[Q_1^{a_1}.Q_2^{b_1}.Q_3^{c_1}.........Q_m^{m_1}.Q_{m+1}\right] \\
[\pi_2] &= \left[Q_1^{a_2}.Q_2^{b_2}.Q_3^{c_2}.........Q_m^{m_2}.Q_{m+2}\right] \\
[\pi_{n-m}] &= \left[Q_1^{a_{n-m}}.Q_2^{b_{n-m}}.Q_3^{c_{n-m}}.........Q_m^{m_{n-m}}.Q_{m+1}\right]
\end{aligned} \tag{12}$$

To make use of any one of the equations of Eq. (12) for finding the specific fom1 of the corresponding Eq. (12), each of the [Q]s is replaced by the known dimensional equivalent for it, in terms of whatever set of "k" fundamental units (such as mass, length, time, etc.). The resulting equation contains the "k" independent fundamental units, and since both members are of zero dimensions, the exponent of each unit must vanish. The "k" independent linear equations are obtained which suffice to determine the "k" exponents. It is still however one arbitrary choice left which is sometimes convenient to make use of it.

As in the above relations, each of them is dimensionless and the exponents a, b, c, d... m, are defined by the dimensions homogeneity. Lastly, as shown in Eq. (13), the general relation for the phenomenon can be obtained by specifying any of the terms as a function of the others (Buckingham 1914; De Rosa et al. 2016).

$$\begin{aligned}
\pi_1 &= f_1(\pi_2, \pi_3..........\pi_{n\text{-}m}) \\
\pi_2 &= f_2(\pi_1, \pi_3..........\pi_{n\text{-}m})
\end{aligned} \tag{13}$$

A matrix of dimensional analysis is used to define the exponents for determining the dimensionless model. In this method, only length L, mass M, and time T as fundamental dimensions are considered. The relation for dependent variable Q_1 which depends on variables {Q_2, Q_3, and Q_4 …Q } is considered as Eq. (14).

$$[Q_i] = L^{l_i} M^{m_i} T^{t_i} \qquad i = 1, \ldots, n \tag{14}$$

The dimension vector for each Q_i can be defined by P_i in Eq. (15).

$$P_i = \begin{pmatrix} l_i \\ m_i \\ t_i \end{pmatrix} \qquad i = 1, \ldots, n \tag{15}$$

And the dimensional matrix is

$$A = \begin{bmatrix} l_1 & l_2 & \ldots & l_n \\ m_1 & m_2 & \ldots & m_n \\ t_1 & t_2 & \ldots & t_n \end{bmatrix} \qquad i = 1, \ldots, n \tag{16}$$

From the above relations, it can be concluded for Q = (Q_1, Q_2, Q_3 …Q) that the number of dimensionless quantities can be derived by Eq. (17).

$$\mathrm{k} + 1 = \mathrm{n} + 1 - \text{ rank (A)} \tag{17}$$

In other words, for "k" linearly independent solution, rank (A) = n – k and AZ = 0 which is denoted by Z^1, Z^2... Z^k. If a j column vector of a, is supposed to be the dimension vector of Q, and **y** as "i" column vector, Eq. (18) represents the relation for defining exponent y. Also, it can be inferred from the reduction factor "k" and rank (A) that the reduction factor can be described as the maximum rank (A) that $|A| \neq 0$.

$$Ay = -a \tag{18}$$

As it was mentioned before, the relation Q = (Q_1, Q_2, Q_3 …Q) can be simplified by Eq. (19).

$$\pi = f(\pi_1, \pi_2..........\pi_n) \tag{19}$$

From Eq. (19), Eq. (20) can be considered for Q.

$$\pi = QQ_1^{y_1} Q_2^{y_2} Q_3^{y_3}......Q_n^{y_n} \tag{20}$$

For every dimensionless quantity π_i

$$\pi_i = Q_1^{Z_1^i} Q_2^{Z_2^i}......Q_n^{Z_n^i} \qquad i = 1, ..., k \tag{21}$$

So, determining the exponent "y_i" by Eq. (18) can lead to

$$u = Q_1^{-y_1} Q_2^{-y_2}......Q_n^{-y_n} f(\pi_1, \pi_2..........\pi_k) \tag{22}$$

Dimensionless groups

Buckingham pi theorem is based on the dimensionless groups so that dimensionless groups can be created after determining the effective independent parameters on the

Table 1 Dimensional matrix for independent and dependent parameters

Dimension	F	D	d	t	r_p	r_d	U	μ
M	1	0	0	0	0	0	1	0
L	1	1	1	1	1	1	− 1	0
T	− 2	0	0	0	0	0	− 2	0

response variable, considering the similarity law and the principle of dimensional homogeneity. According to the similarity law, the prototype and physical model for dimensional analysis should be in complete similarity which means that all relevant dimensionless parameters for the conditions of the process have the same corresponding values for the model and the prototype. Since the deep drawing process is considered as a quasi-static process, the dynamic similarity was passed up in this research. In this research, the prototype (small scale) and physical model (large scale) were drawn in the same velocity (similar strain ration). So the kinematic similarity is considered to be achievable. Therefore, it is necessary for this research to obtain geometric similarity. To achieve comparable results during scale changing, all process parameters that can affect the final process quality including geometrical parameters such as blank and punch dimensions, punch and die edge radius, drawing depth, sheet thickness, and the gap between the punch and die must be in the same scale. According to the similarity law, the same material with the same mechanical and thermal properties should be considered for the small and large scales. It is mentioned in (Tan 2011; Zare et al. 2012) that the material properties for small and large scales should be considered equal if the kinematic similarity is achievable. It means that in addition to the same ratio for velocities (kinematic similarity), the homologous particles can lie at homologous points at homologous times if the mechanical properties such as density, young modulus, and tension strength are the same for the prototype (small scale) and physical model (large scale). On the other hand, all parameters affecting the failure or damage of the material should be considered the same to investigate tearing in the original sample (De Rosa et al. 2016).

Finally, the relationship between the drawing force and the effective parameters can be determined in Eq. (23).

$$F = f\left(D, d, t, r_p, r_d, U, \mu\right) \tag{23}$$

where "D" is the blank diameter, "d" is the punch diameter, punch, "t" is the sheet thickness, "r_p" is the punch edge radius, "r_d" is the die edge radius, "U" is the tensile strength of sample material, and "μ" is friction coefficient. The total number of independent and dependent variables is n = 8 which are presented in Table 1. Considering the reduction factor "$k = n - m$," 5 dimensionless groups can be derived by the number of main dimension m = 3. However, given that the determinant of the dimensional matrix must be zero and that the current dimensional matrix with 3 primary dimensions cannot be zero, two primary dimensions must be used to determine the number of the dimensionless group (Tan 2011). As a result, 6 dimensionless groups were determined.

Since the friction coefficient is generally a dimensionless factor, then one of the dimensionless groups can be friction coefficient which is presented in Eq. (24).

$$\pi_6 = \mu \tag{24}$$

The calculation steps to describe the indices that determine the first dimensionless number have been defined in Eq. (25) to simplify the understanding of the method used.

$$\pi_1 = FD^{\alpha}U^{\beta} \tag{25}$$

According to this relationship, the input and output variables are written in the first step. In the second step, the relation π_1 shown in Eq. (26) in terms of the main dimensions.

$$\pi_1 = \left(MLT^{-2}\right)\left(L^{\alpha}\right)\left(ML^{-1}T^{-2}\right)^{\beta} \tag{26}$$

In the third step, in view of dimensionless number, the homogeneity shown in Eq. (27).

$$\left(MLT^{-2}\right)\left(L^{\alpha}\right)\left(ML^{-1}T^{-2}\right)^{\beta} = M^0L^0T^0 \tag{27}$$

In accordance with Eq. (28), in the next step, the equation of dimensional equality is written as

Table 2 Dimensionless groups

1	$\pi_1 = F/(D^2.U)$	$\pi_2 = t/D$	$\pi_3 = d/D$	$\pi_4 = R/D$	$\pi_5 = r/D$	$F/(D^2.U) = f(t/D,d/D,R/D, r/D,\mu)$
2	$\pi_1 = F/(d^2.U)$	$\pi_2 = t/d$	$\pi_3 = D/d$	$\pi_4 = R/d$	$\pi_5 = r/d$	$F/(d^2.U) = f(t/d,D/d,R/d, r/d,\mu)$
3	$\pi_1 = F/(t^2.U)$	$\pi_2 = D/t$	$\pi_3 = d/t$	$\pi_4 = R/t$	$\pi_5 = r/t$	$F/(t^2.U) = f(D/t,d/t,R/t, r/t,\mu)$
4	$\pi_1 = F/(R^2.U)$	$\pi_2 = D/R$	$\pi_3 = d/R$	$\pi_4 = t/R$	$\pi_5 = r/R$	$F/(R^2.U) = f(D/R,d/R,t/R, r/R,\mu)$
5	$\pi_1 = F/(r^2.U)$	$\pi_2 = D/r$	$\pi_3 = d/r$	$\pi_4 = t/r$	$\pi_5 = R/r$	$F/(R^2.U) = f(D/R,d/R,t/R, R/r,\mu)$

Table 3 Properties of St12

Strength in tension (MPa)	350
Modulus of elasticity (GPa)	200
Yield strength (MPa)	210
Density (kg/m3)	7800

$$\begin{aligned} &1+\beta=0 \\ &1+\alpha-\beta=0 \\ &-2-2\beta=0 \end{aligned} \tag{28}$$

Finally, dimensionless number π_1 which contains drawing force is obtained by Eq. (29).

$$\pi_1=\frac{F}{(D^2U)} \tag{29}$$

The dimensionless groups obtained by Buckingham pi theorem are shown in Table 2.

Table 4 Input and output parameters of the simulation

No.	*d* (mm)	*D* (mm)	*t* (mm)	*R* (mm)	*r* (mm)	*μ*	*F* (*kN*)
1	40	80	0.1	6.5	5	0.1	4.625
2	120	240	0.3	19.5	15	0.1	42
3	200	400	0.5	32.5	25	0.1	112
4	40	80	0.1	6.5	5	0.05	3.95
5	120	240	0.3	19.5	15	0.05	35.5
6	200	400	0.5	32.5	25	0.05	98.7
7	40	80	0.1	6.5	5	0.2	6.24
8	120	240	0.3	19.5	15	0.2	56
9	200	400	0.5	32.5	25	0.2	156
10	33.3	66.6	0.08	5.4	4.2	0.1	4.105
11	50	100	0.12	8.1	6.3	0.1	9.235
12	75	150	0.18	12.15	9.5	0.1	37
13	33.3	66.6	0.08	5.4	4.2	0.05	3.435
14	50	100	0.12	8.1	6.3	0.05	7.728
15	75	150	0.18	12.15	9.5	0.05	31
16	33.3	66.6	0.08	5.4	4.2	0.2	5.72
17	50	100	0.12	8.1	6.3	0.2	12.87
18	75	150	0.18	12.15	9.5	0.2	51.48
19	25.6	51.2	0.06	4.15	3.23	0.1	27.745
20	38.4	76.8	0.09	6.22	4.85	0.1	80.183
21	57.6	115.2	0.13	9.34	7.27	0.1	202.261
22	25.6	51.2	0.06	4.15	3.23	0.05	25.382
23	38.4	76.8	0.09	6.22	4.85	0.05	73.354
24	57.6	115.2	0.13	9.34	7.27	0.05	534.75
25	25.6	51.2	0.06	4.15	3.23	0.2	33.07
26	38.4	76.8	0.09	6.22	4.85	0.2	95.572
27	57.6	115.2	0.13	9.34	7.27	0.2	241.08

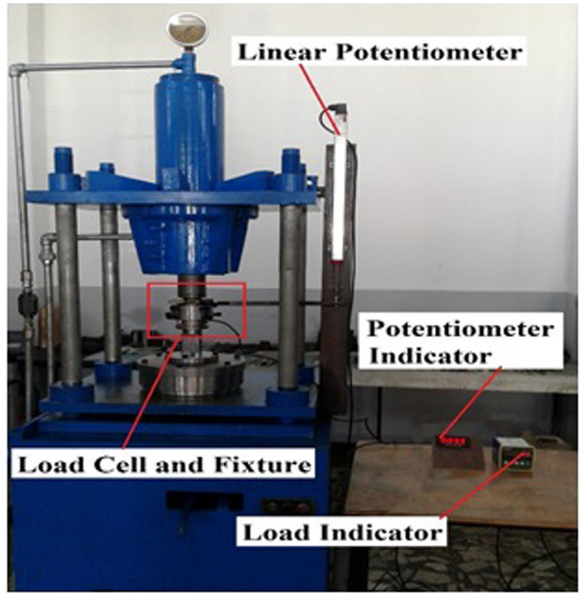

Fig. 1 Experimental setup

Simulation

In this section, to determine the drawing force at tear moment, the deep drawing process for the St12 carbon steel plate with the standard number DIN 1.0374 was simulated by a 3D model in ABAQUS commercial software. Table 3 shows the properties of St12 sheet metal (Fereshteh-Saniee and Montazeran 2003). The finite element model based on the explicit solving method was considered one-quarter of the model to reduce the computational cost. Deformable shell model with a four-node shell element was considered for sheet and the analytical rigid shell was defined for other components. By applying boundary conditions, the matrix and blank


Fig. 2 Precision digital thickness gauge

Table 5 Process factors and their levels

	d	*D*	*t*	*R*	*μ*
1	30	54	0.5	3	0.2
2	60	108	0.5	6	0.18
3	120	216	0.5	12	0.15
4	30	54	1	3	0.2
5	60	108	1	6	0.18
6	120	216	1	12	0.15
7	30	54	2	3	0.2
8	60	108	2	6	0.18
9	120	216	2	12	0.15

holder movements were limited and the edge of the sheet was considered to be completely constant. Also, 40% reduction in thickness as a tear criterion and friction coefficient 0.1 was considered for all conditions. Input parameters and drawing force at the tearing moment are presented in Table 4.

Experimental setup

The experimental tests were carried out on 25-ton capacity hydraulic press. The material used for experiments is St12 stainless steel with standard number DIN 1.0374 which is so applicable in various industries. The compositions (wt %) of this sheet metal were S (< 0.04), N (< 0.007), P (< 0.04), Mn (0.4), and C (< 0.1) (Fereshteh-Saniee and Montazeran 2003). In this deep drawing mold, to save costs and machining time, die and blank holder was manufactured as inserted part. To measure the drawing force, the 33-ton capacity load cell manufactured by CELLTEC company was used which was fixed on the press ram by a fixture. The indicator with 0.00005 external resolution manufactured by the SEWHA company was used to display the load. The linear potentiometer with a 0.5% error manufactured by OPKON company was used to measure the height of the drawn cup. All setup for load cell and the linear potentiometer is shown in Fig. 1. To determine the effective friction coefficient, thickness distribution of a drawn sheet was used which was measured by precision digital thickness gauge with 1-μm resolution and 5-μm measurement precision manufactured by GMAG company which is shown in Fig. 2.

Experimental design

All the effective geometrical parameters are shown in Table 5. The levels of these three scales were selected according to our laboratory experiences. To ensure the precision of results, each experiment was measured three times and the mean value for the output was considered. The friction coefficient in experimental tests was determined by comparing thickness distribution in experimental tests with simulation. For this purpose, sheet metal thickness was measured in eight locations concerning their distance to the center of the sheet which is shown in Fig. 3. Then, the effective friction coefficient on each sample was determined by comparing the thickness in experimental and simulation cases which is shown in Fig. 4.

Dimensionless model

All possible dimensionless combinations have been analyzed using a developed SPSS statistical software by regression modeling. For this propose, every double combination in each dimensionless group has been evaluated by analysis of variance (ANOVA). Dimensionless combinations, considering the correlation coefficients R^2, are given in Table 6. The correlation coefficient is obtained by dividing the regression sum of squares (SSR) into the total sum of squares (SST) according to Eq. (30). Considering R^2, it was determined that combination 1 has a higher correlation coefficient than others.

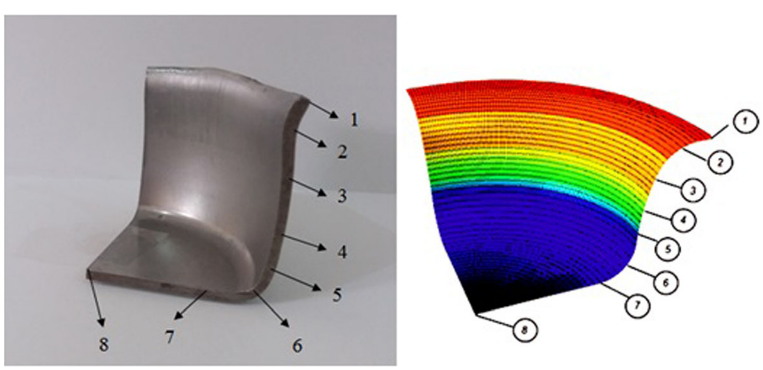

Fig. 3 Location of measure points in experimental and simulation case

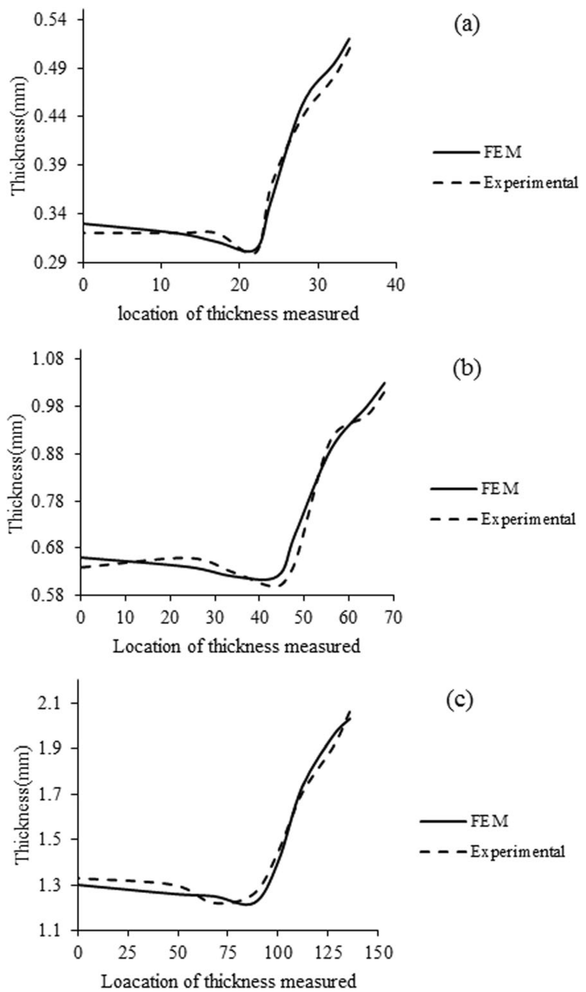

Fig. 4 Thickness distribution in experimental and simulation cases. **a** Scale 1:1. **b** Scale 2:1. **c** Scale 4:1

$$R^2 = \frac{SSR}{SST} \tag{30}$$

However, it should be noted that despite the high correlation coefficient for combinations 1 and 2, the friction coefficient as an important parameter for predicting drawing force which is considered as dimensionless parameter π_6 has not any role in predicting drawing force. For this reason, to specify a precision and significant model by considering effective dimensionless parameters, the stepwise regression method was used.

Table 6 Dimensional combinations with the best correlation

	Combination	R^2	Regression equation
1	$\pi_1 = f(\pi_2,\pi_3)$	0.9	$0.0011 \times \pi_2^{0.011} \times \pi_3^{0.8}$
2	$\pi_1 = f(\pi_2,\pi_4)$	0.9	$2.126 \times \pi_2^{1.366} \times \pi_4^{2.795}$
3	$\pi_1 = f(\pi_2,\pi_5)$	0.86	$2.109 \times \pi_2^{1.358} \times \pi_5^{2.776}$
4	$\pi_1 = f(\pi_2,\pi_6)$	0.89	$0.63 \times \pi_2^{1.01} \times \pi_6^{0.26}$

The stepwise regression method evaluates all the parameters, but unlike the general regression modeling methods, there is not any interference in the model by insignificant factors. In other words, the participation of the factor in the final model is ignored if there is no significant influence on that parameter. The criterion for adding or removing a variable at any step is usually expressed in terms of a partial *F*-test. In this test, f_{in} is defined as the value of the *F*-random variable for adding a variable to the model and f_{out} is defined as the value of the *F*-random variable for removing a variable from the model which it must be $f_{\text{in}} \geq f_{\text{out}}$. Stepwise regression begins by forming a one-variable model using the x_1 regressed variable that has the highest correlation with the response variable "*Y*." In the second step, the remaining K-1 candidate variables are examined, using the partial *F*-statistic factor which is shown in Eq. (31) and it provided that $f_j \geq f_{\text{in}}$.

$$F_j = \frac{SS_R\left(\beta_j|\beta_1,\beta_0\right)}{MS_E\left(x_j,x_1\right)} \tag{31}$$

In Eq. (31) MS_E (x_j, x_1) denotes the mean square of error for the model containing both x_1 and x_{j}. Suppose that this procedure indicates that x_2 should be added to the model. Now, the stepwise regression algorithm determines whether the variable x_1 added at the first step should be removed. This is done by calculating the *F*-statistic based on Eq. (31) which is shown in Eq. (32).

$$F_j = \frac{SS_R(\beta_1|\beta_2,\beta_1)}{MS_E(x_1,x_2)} \tag{32}$$

If the calculated value $f_1 < f_{\text{out}}$, the variable x_1 is removed; otherwise, it is retained, and it would be attempted to add a regressor to the model containing both x_1 and x_2. In general, at each step, the set of remaining candidate regressors is examined, and the regressor with the largest partial *F*-statistic is entered, provided that the observed value of *f* exceeds f_{in}. Then the partial *F*-statistic for each regressor in the model is calculated, and the regressor with the smallest observed value of *F* is deleted if the observed $f < f_{\text{out}}$. The procedure continues until no other regressors can be added to or removed from the model. The schematic of the stepwise regression process is presented in Fig. 5. At long last, in the wake of examining every one of the dimensionless factors in several steps, the best-fitted model is extracted which is composed of effective parameters. The correlation coefficient of the acquired model is more trustworthy which is given in ANOVA Table 7. When the value of R^2 is close to 1, it indicates that

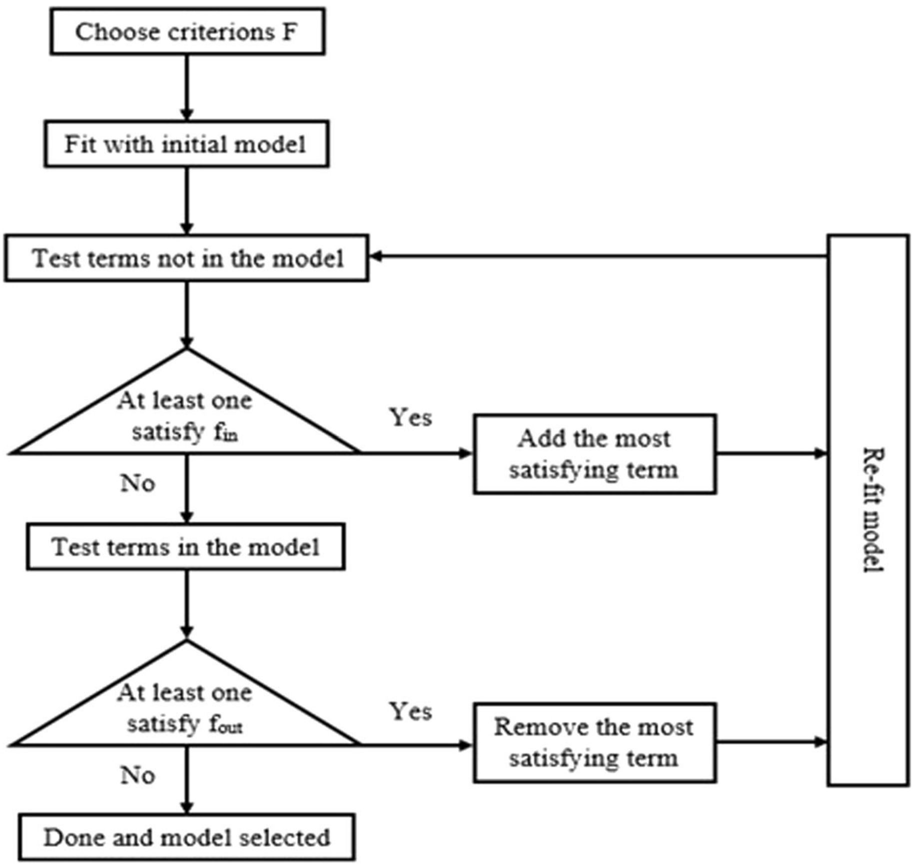

Fig. 5 Schematic of the stepwise regression method

there is a strong relationship between input and output variables. In analyzing the regression model, it should be noted that the R^2 coefficient increases by adding additional variables or higher levels to the model. Therefore, models with greater R^2 may be weak in predicting or estimating outputs for new inputs, despite good fit for existing data. Therefore, the adjusted correlation coefficient R^2_{adj} shown in Eq. (33), which can prevent the inclusion of unnecessary factors, can provide more accurate analysis in the regression model. In Eq. (33), n is the number of experiments and p is the number of factors. When the values of R^2 and R^2_{adj} are very different, it means that additional variables have been added to the model. In addition to the residuals and correlation coefficient, another parameter that is used to measure the suitability of the regression model is the parameter P value. $P < \alpha$ indicates the appropriateness of the regression model, in which α is a confidence level and is usually considered to be between 5 and 10%. This means that an error of α% is allowed in the experiment (Montgomery 2003).

Table 7 ANOVA results for the dimensionless model by stepwise regression

Source	Sum of squares	Degree of freedom	Mean square	*F* value	*P* value	% contribution
Model	0.956	10	0.0956	7.468	0.008	
π_2	0.412	2	0.206	16.093	0.0102	32.07
π_3	0.312	2	0.156	12.187	0.0145	24.28
π_4	0.227	2	0.113	8.828	0.0175	17.61
π_5	0.036	2	0.018	1.406	0.346	2.81
π_6	0.246	2	0.123	9.6093	0.0135	19.15
Lack of fit	0.066	5	0.0132	1.031	0.495	2.05

$R^2 = 0.91$, $R^2_{adj} = 0.93$, $R^2_{pred} = 0.92$

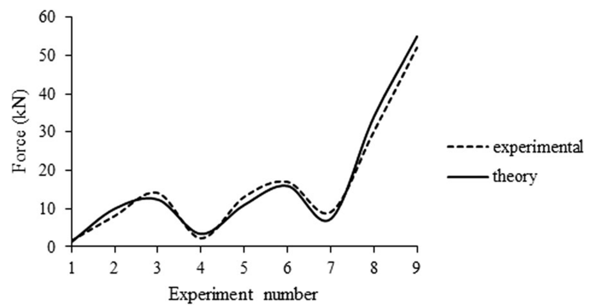

Fig. 6 Tearing forces obtained from the experimental results and the dimensionless equation

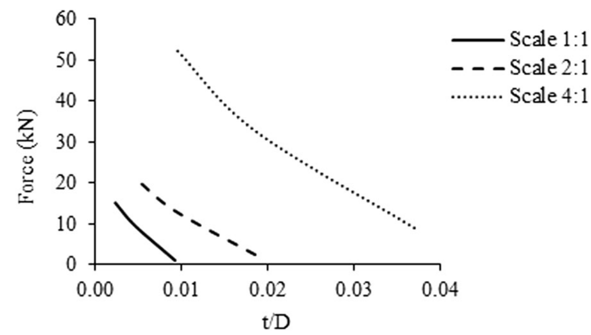

Fig. 8 Effect of the dimensionless parameter on drawing force

$$R^2_{\text{adj}} = 1 - \frac{n-1}{n-p}\left(1-R^2\right) \quad (33)$$

The correlation coefficient of the obtained model with stepwise regression is more reliable which is given in ANOVA Table 7. Consequently, as shown in Eq. (34), dimensionless numbers π_2, π_3, π_4, and π_6 from group 1 remained in the regression model. The associated *P* value of less than 0.05 for the model (i.e., α = 0.05 or 95% confidence level) indicates that the model terms are statistically significant. It can be seen in Eq. (34) that dimensionless number π_4 has been eliminated from the final model because of its insignificant *P* value (*P* value > 0.05). Also, it is inferred from the ANOVA table that dimensionless number π_2 is the most significant factor that affects the dimensionless response π_1 which contains the tearing force. Another factor in Table 7 is the contribution percent that is resulted by *F* value. Larger values of contribution indicate that changing in the corresponding parameter will have a large impact on the response variable (Montgomery 2003). In other words, more contribution to an input parameter leads to more effective response factor. It can be seen from Table 7 that dimensionless parameter π_2 that specifies the ratio of t/D is the most contributed and dimensionless parameter π_5 that calculates the r/D has the lowest contribution to the response factor π_1 which contains drawing force.

$$\pi_1 = 0.121 \times {\pi_2}^{0.688} \times {\pi_3}^{0.4} \times {\pi_4}^{1.397} \times {\pi_6}^{0.38} \quad (34)$$

Results and discussion

Confirmation run

According to Table 5, in order to verify the developed model, nine confirmation experiments were performed and the results of the experimental tests were compared with the proposed dimensionless model. The input parameters have been selected different from those in simulation to carefully evaluate the proposed dimensionless model. It is inferred from Fig. 6 that the developed dimensionless model with 8.8% mean error has good precision in the prediction of drawing force in deep drawing of round cups. The round drawn cups are shown in Fig. 7 for three scales.

Analysis of dimensionless parameters

As indicated by acquired outcomes, comparison of experimental results with Buckingham pi theorem demonstrated that the proposed dimensionless models have high precision to estimate drawing force at tearing

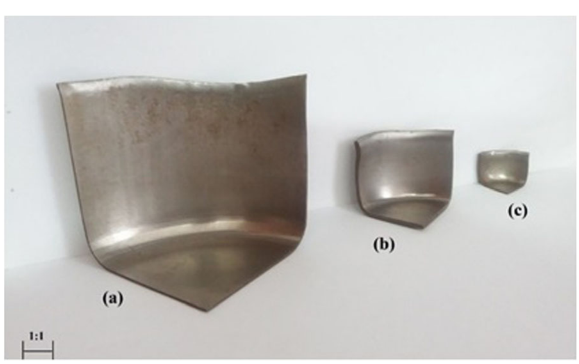

Fig. 7 Round cups. **a** Scale 4:1. **b** Scale 2:1. **c** Scale 1:1

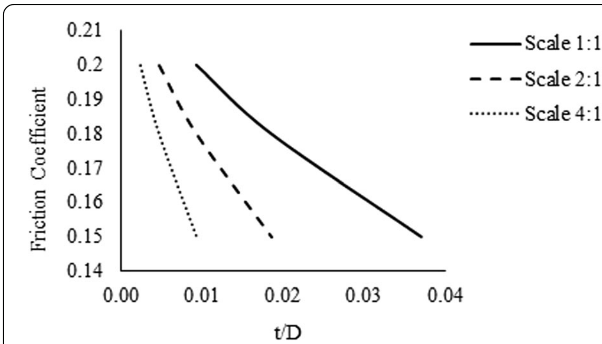

Fig. 9 Geometrical parameters determination by the friction coefficient

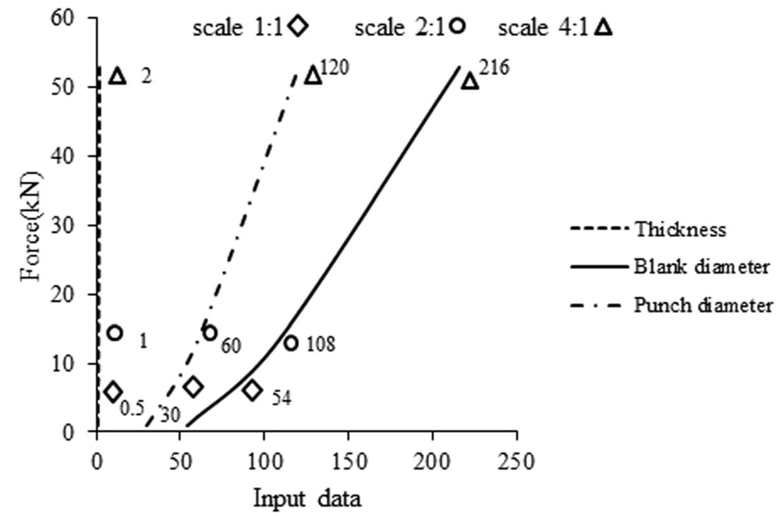

Fig. 10 Drawing force respect to geometric parameters in different scales based on the proposed dimensionless model

moment without simulation and experimental tests. According to ANOVA results, the π_2 dimensionless parameter, which indicates proportion t/D, can be utilized as effective dimensionless parameters to evaluate drawing force from small to large scale. As shown in Fig. 8, changing in drawing force in different scales requires changing in geometric proportions to produce the flawless part. For example, when the drawing force for a flawless part in scale 1:1 is measured, according to the similarity law, if the blank thickness for the next scale is doubled, the blank diameter is changed similarly for maintaining the proportion of t/D. On the other hand, because of the high effect of friction coefficient on drawing force, it can be considered for determining the appropriate geometric parameters. As can be seen in Fig. 9, the value of t/D can be detected in three different scales by friction coefficient.

According to similitude, in order to produce flawless cups in different scales, scale changing that leads to drawing force changing requires changing in geometric ratios. For example, when the force for a flawless piece of scale 1:1 is 9.8 kN, if the thickness of the next scale changes from 1 to 2 mm, according to the similarity law, to maintain the ratio of thickness to the sheet diameter and to ensure that the sheet is properly drawn, the diameter of the blank should increase by 2 times. As shown in Fig. 10, changing blank diameter from 108 to 150 mm, 60 to 83 mm for punch diameter, and 1 to 1.4 mm for sheet thickness leads to about 30 kN drawing force which it concludes that it is well estimated based on the proposed dimensionless model in comparison with 33 kN resulted by experiments. So, it can be expressed that for the same material in larger scales, the value of geometric parameters and drawing force can be estimated with good accuracy by smaller geometry based on the proposed dimensionless model.

Table 8 Process factors for conducting verification runs

Scale	$\frac{t}{D}$			
1:1	0.001	0.006	0.01	0.02
2:1	0.004	0.01	0.025	0.04
4:1	0.01	0.03	0.05	0.07

Generalization

In order to investigate the generalizations of proposed dimensionless analysis and dimensionless models, the parameter t/D was considered for generalizing by simulation. The values of these parameters were selected out of the range of experimental design which is appeared in Table 8. The predicted values for verification in all three scales are precisely along with the trend of graphs as shown in Fig. 11. Likewise, as indicated by the confirmation, it is possible to generalize the analysis with good precision in each scale, to estimate drawing force and appropriate geometric parameters.

Conclusion

The current investigation mainly involves a dimensionless analysis based on the Buckingham pi theorem which is developed by simulation and experimental design for deep drawing of the round cup. The empirical models were derived by stepwise regression to predict drawing force and evaluation of dimensionless factors in different geometrical scales. To derive these models, some input and output data are used for simulation. Derived empirical models were evaluated by experimental data. Finally, in order to show the ability of the proposed models to predict drawing force, generalization by FEM simulation

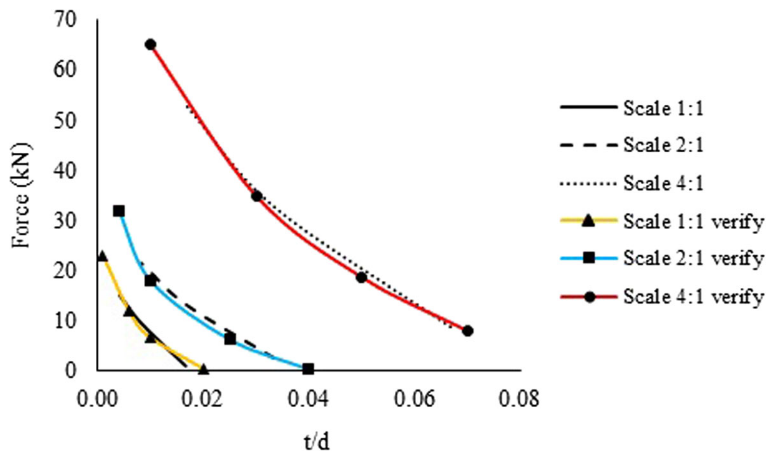

Fig. 11 Generalization of proposed dimensionless model

and some out of scope data was done. From the research study, the following conclusions can be counted herein:

1- Similarity conditions and Buckingham pi theorem have shown that various dimensionless groups, each containing different dimensionless parameters, can be used to predict deep drawing force.
2- The correlation coefficients of dimensionless models for each group which were obtained by simulation showed that there is no adequate confidence for all the specified dimensionless on drawing force prediction.
3- It was determined that dimensionless parameters with more than 90% correlation coefficient can be valid for predicting drawing force. The results of ANOVA and the validation experiments confirm that the proposed dimensionless model shows good accuracy with an average error of less than 9% in predicting drawing force for round cups.
4- It was shown from the ANOVA results that dimensionless parameters t/D, d/D, R/D, and friction coefficient are majorly significant. It was also shown that dimensionless ratio t/D is the most dominant dimensionless parameters for estimating the drawing force.
5- The results of the dimensionless analysis and proposed dimensionless models have an excellent capability for generalizing which was verified by simulation. Therefore, it can be said that geometric values in larger scales can be estimated with good precision by smaller scales for the same material.

Finally, it is worth to mention that the dimensional analysis tool applied in the present study has given a general outline of drawing force estimation for correlating the laboratory studies on the actual industrial scale.

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Authors' contributions
SH took on most of the research work, including the theoretical research and modeling, proposal and establishment of the new method and simulation work, and paper writing of the manuscript. ME and MS put forward a great variety of valuable suggestions on some key theory points and assisted with the theory researching and method validity so that the research work can be carried out smoothly. All authors read and approved the final manuscript.

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